

The law of evolution of energy density in the universe imposed by quantum cosmology and its consequences

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Abstract

The quantum model of homogeneous and isotropic universe filled with the uniform scalar field is considered. The time-independent equation for the wave-function has the solutions which describe the universe in quasistationary states. The evolution of the universe is realized in the form of transitions between such states. In the first stage in the early universe scalar field slow rolls into a vacuum-like state with a vanishing energy density. In the second stage this field begins to oscillate near the minimum of its potential energy density and becomes a source of creation of matter/energy in the universe. The quantum model predicts effective inverse square-law dependence of the mean total energy density $\bar{\rho}$ on the expectation value of cosmological scale factor $\langle a \rangle$ where the averaging is performed over the state with large quantum numbers. Such a law of decreasing of $\bar{\rho}$ during the expansion of the universe allows to describe the observed coordinate distances to type Ia supernovae and radio galaxies in the redshift interval $z = 0.01 - 1.8$. A comparison with phenomenological models with the cosmological constant (Λ CDM) and with zero dark energy component ($\Omega_M = 1$) is made. It is shown that observed small deviations of the coordinate distances to some sources from the predictions of above mentioned simple quantum model can be explained by the fluctuations δa of the scale factor about the average value $\langle a \rangle$. These fluctuations can arise due to finite widths of quasistationary states in the early universe. During expansion the fluctuations δa grow with time and manifest themselves in the form of observed relative increase or decrease of coordinate distances. The amplitudes of fluctuations $\delta a / \langle a \rangle$ calculated from observed positions of individual supernovae are in good agreement with their estimations in quantum theory. Proportionality of the average value $\langle a \rangle$ to total quantity of matter/energy in the universe on the one hand and to its age on the other hand predicted by the quantum model agrees with the present-day cosmological observables. Possible consequences from the conclusions of quantum theory are discussed.

1 Introduction

The observed faintness of the type Ia supernovae (SNe) at the high redshift [1, 2] attracts cosmologists' attention in connection with the hypothesis of an accelerating expansion of the present-day universe proposed for its explanation [1, 2, 3, 4, 5]. Such a conclusion assumes that dimming of the SNe Ia is hardly caused by physical phenomena non-related to overall expansion of the universe as a whole, such as unexpected luminosity evolution, effects of contaminant gray intergalactic dust, gravitational lensing, or selection biases (see Ref. [6] for review and [7] about the absorption of light by metallic dust). Furthermore it is supposed that matter component of energy density in the universe ρ_M , which includes visible and invisible (dark) baryons and dark matter, varies with the expansion of the universe as a^{-3} (i.e. it has practically vanishing pressure, $p_M \approx 0$), where a is the cosmological scale factor, while mysterious cosmic fluid (so-called dark energy [8, 9]) is describes by the equation of state $p_X = w_X \rho_X$, where $-1 \leq w_X \leq -\frac{1}{3}$ [3, 4]. Parameter w_X can be constant, as e.g. in the models with the cosmological constant, $w_X = -1$, (Λ CDM-models) [3, 4, 6], or may vary with time as in the rolling scalar field scenario (models with quintessence) [4, 5]. Even if regarding baryon component one can assume that it decreases as a^{-3} (pressure of baryons may be neglected due to their relative small amount in the universe), for dark matter (whose nature and properties can be extracted only from its gravitational action on ordinary matter) such a dependence on the scale factor may not hold in the universe taken as a whole (in contrast to local manifestations e.g. in large-scale structure formation, where dependence a^{-3} may survive). Since the contribution from all baryons into the total energy density does not exceed 4% [10], the evolution of the universe as a whole is determined mainly by the properties of dark matter and dark energy. The models of dark energy [3, 4, 8] show explicitly unusual behaviour of this component during the expansion of the universe.

According to modern astrophysical data the mean energy density in the present-day universe is estimated as $\rho_0 \sim 10^{-29} \text{ g cm}^{-3}$ [10], the mass of its observed part is $M_0 \sim 10^{80} \text{ GeV}$, while its radius of curvature is $a_0 \sim 10^{28} \text{ cm}$ [11, 12, 13]. The age of the universe equals to $t_0 \sim 10^{17} \text{ s}$ [4, 10, 14]. At the same time the dimensionless age parameter is $H_0 t_0 \sim 1$, where H_0 is the present-day value of the Hubble constant [4, 6]. If one expresses these values in modified Planck units, where length and density are measured in $l_P = \sqrt{2G/(3\pi)}$ and $\rho_P = 3/(8\pi G l_P^2)$ respectively [15, 16], the simple relation between the observed parameters of the present-day universe will be revealed

$$M_0 \sim a_0 \sim t_0 \sim 10^{61}, \quad (1)$$

while its total energy density will be

$$\rho_0 \sim \frac{1}{a_0^2} \sim \frac{1}{t_0^2} \sim 10^{-122}. \quad (2)$$

If these relations will not be considered as an accidental coincidence realized for unknown reason at the present epoch (as in the hypothesis of fine tuning or compensation of the cosmological constant [4, 17]), it is reasonable to assume the existence of the epoch, when the energy density in the universe effectively decreases as a^{-2} . In the present paper we pay attention to the fact that exactly such a dependence of the mean energy density $\bar{\rho}$ on the average $\langle a \rangle$ in the state with large quantum numbers which describes only homogenized properties of the universe is predicted by the quantum model of the homogeneous and isotropic universe proposed in Refs. [15, 16]. We show that the quantum model, where the density $\bar{\rho} \sim \langle a \rangle^{-2}$, allows to explain the observed coordinate distances to SNe Ia and radio galaxies (RGs) in wide redshift range $z = 0.01 - 1.8$. The observed small deviations of the coordinate distances to some sources from the predictions of the model with $\bar{\rho} \sim \langle a \rangle^{-2}$ can be explained by the local manifestations of quantum fluctuations δa of the scale factor about its average value $\langle a \rangle$. These fluctuations produce accelerating or decelerating expansions of space subdomains containing separate sources with high redshift whereas the universe as a whole expands at a steady rate. The amplitudes of fluctuations $\delta a / \langle a \rangle$ calculated from observed positions of individual supernovae are in good agreement with their estimations in quantum theory. We make a comparison with phenomenological Λ CDM-model and the model with zero dark energy component ($\Omega_M = 1$). Possible consequences from the conclusions of quantum theory are discussed.

2 Quantum model

Just as in ordinary quantum nonrelativistic and relativistic theories one can assume that the problem of evolution and properties of the universe as a whole in quantum cosmology should be reduced to the solution of the functional partial differential equation determining the eigenvalues and the eigenstates (in space of generalized variables, whose roles are played by the metric tensor components and matter fields) of some hamiltonian-like operator. In such an approach the Wheeler-DeWitt equation [18, 19] will be the equation with zero eigenvalue. As it is shown in Refs. [15, 16] the homogeneous, isotropic and spatially closed universe filled with primordial matter in the form of the uniform scalar field ϕ with some potential energy density $V(\phi)$ is described by the equation (all in units of $l_P = 1$, $\rho_P = 1$)

$$(-\partial_a^2 + a^2 - a^4 \hat{\rho}_\phi - E) \psi_E = 0, \quad (3)$$

where the operator

$$\hat{\rho}_\phi = -\frac{2}{a^6} \partial_\phi^2 + V(\phi), \quad (4)$$

corresponds to the energy density of the scalar field in classical theory (cf. e.g. Ref. [4]), while the wavefunction ψ_E is given in (a, ϕ) -space of two variables, the scale factor a and matter field ϕ . The eigenvalue E determines the components of

the energy-momentum tensor

$$\tilde{T}_0^0 = \frac{E}{a^4}, \quad \tilde{T}_1^1 = \tilde{T}_2^2 = \tilde{T}_3^3 = -\frac{E}{3a^4}, \quad \tilde{T}_\nu^\mu = 0 \quad \text{for } \mu \neq \nu. \quad (5)$$

For $E > 0$ it coincides with the energy-momentum tensor of relativistic matter. But for $E < 0$ one cannot associate any physical source with tensor \tilde{T}_ν^μ . We shall consider the case $E > 0$ and call a source determined by the energy-momentum tensor (5) a radiation.

From Eq. (3) it follows the relation for the average values in the state ψ_E normalized in one way or another

$$\left\langle -\frac{1}{a^4} \partial_a^2 \right\rangle = \langle \hat{\rho}_\phi \rangle + \left\langle \frac{E}{a^4} \right\rangle - \left\langle \frac{1}{a^2} \right\rangle. \quad (6)$$

We shall assume that the average value $\langle a \rangle$ determines the scale factor of the universe in classical approximation. Then the relation (6) takes the form of the first Einstein-Friedmann equation in terms of average values (see Appendix A)

$$\left(\frac{1}{\langle a \rangle} \frac{d\langle a \rangle}{dt} \right)^2 = \bar{\rho} - \frac{1}{\langle a \rangle^2}, \quad (7)$$

where

$$\bar{\rho} = \frac{2}{\langle a \rangle^6} \langle -\partial_\phi^2 \rangle + \langle V \rangle + \frac{E}{\langle a \rangle^4} \quad (8)$$

is the mean total energy density, $H = (1/\langle a \rangle) d\langle a \rangle/dt$ is the Hubble constant.

In order to specify the solution of Eq. (3) at given $V(\phi)$, it has to be supplemented by boundary conditions. In the asymptotic region $a^2 \gg 1$ the solution of (3) can be represented in the form [16]

$$\psi_E \sim c^{(-)}(E) \varphi_E^{(-)} + c^{(+)}(E) \varphi_E^{(+)}, \quad (9)$$

where $\varphi_E^{(-)}(a, \phi)$ and $\varphi_E^{(+)}(a, \phi)$ are the wave incident upon the barrier $U = a^2 - a^4 V$ and the outgoing wave respectively, which are considered as the functions of a at given value of the field ϕ . The $c^{(\pm)}(E)$ are some coefficients which depend on E . The boundary condition $c^{(-)}(E) = 0$ selects the outgoing wave from the superposition (9). One can introduce an analog of the S-matrix $S(E) = -c^{(+)}(E)/c^{(-)}(E)$, which will have the poles in the upper half-plane of the complex plane of E at $E = E_n + i\Gamma_n$. These values describe the universe in n -th quasistationary state with the parameters $E_n > 0$ (position of the level) and $\Gamma_n > 0$ (its width), $n = 0, 1, 2, \dots$ (number of the state) [15, 16]. In a wide variety of quantum states of the universe, described by Eq. (3), quasistationary states are the most interesting, since the universe in such states can be characterized by the set of standard cosmological parameters [16]. The wavefunction of the quasistationary state as a function of a has a sharp peak and it is concentrated mainly in the region limited by the barrier U . Therefore

following Ref. [20] one can introduce some approximate function which is equal to exact wavefunction inside the barrier and vanishes outside it. This function can be normalized and used in calculations of expectation values. Such an approximation does not take into account exponentially small probability of tunneling through the barrier U . It is valid for calculation of observed parameters within the lifetime of the universe, when the quasistationary states can be considered as stationary ones with $E = E_n$ (cf. e.g. Ref. [21]).

The quantum state of the universe depends on form and value of the potential $V(\phi)$. Just as in classical cosmology which uses a model of the slow-roll scalar field [13, 22] in quantum theory based on Eq. (3) it makes sense to consider a scalar field ϕ which slowly evolves (in comparison with a large increase of the average $\langle a \rangle$) into a vacuum-like state with $V(\phi_{vac}) = 0$ from some initial state ϕ_{start} , where $V(\phi_{start}) \sim \rho_P$. The latter condition allows us to consider the evolution of the universe in time in classical sense. Reaching the state ϕ_{vac} the field ϕ begins to oscillate about the equilibrium vacuum value due to the quantum fluctuations. Here the potential $V(\phi)$ can be well approximated by the expression

$$V(\phi) = \frac{m^2}{2} (\phi - \phi_{vac})^2, \quad (10)$$

where $m^2 = (d^2V/d\phi^2)_{\phi_{vac}} > 0$. The oscillations in such a potential well can be quantized. The spectrum of energy states of the field ϕ obtained here has a form: $M = m(s + \frac{1}{2})$, where m is a mass (energy) of elementary quantum excitation of the vibrations of the scalar field, while s counts the number of these excitations. The value M can be treated as a quantity of matter/energy in the universe.

3 Mean energy density

The solution of Eq. (3) for the states of the universe with large quantum numbers, $n \gg 1$ and $s \gg 1$, has a form

$$\psi_E = \varphi_n(a) f_{ns}(\phi), \quad E = 4\langle a \rangle [\langle a \rangle - M], \quad (11)$$

where

$$\varphi_n(a) = \frac{1}{\sqrt{\langle a \rangle}} \cos \left(\sqrt{2N+1} a - \frac{N\pi}{2} \right), \quad (12)$$

$$f_{ns}(\phi) = \left[\frac{1}{3 \langle (\phi - \phi_{vac})^2 \rangle} \right]^{1/4} \cos \left(\sqrt{M(2N)^{3/2}} (\phi - \phi_{vac}) - \frac{s\pi}{2} \right). \quad (13)$$

Here $N = 2n + 1$ defines the number of elementary quantum excitations related to the vibrations of geometry with Planck masses in n -th state of the universe [23, 24], while the functions $\varphi_n(a)$ and $f_{ns}(\phi)$ are normalized by the conditions

$$\int_0^{a_c} da \varphi_n^2(a) = 1, \quad \int_{\phi_-}^{\phi_+} d\phi f_{ns}^2(\phi) = 1, \quad (14)$$

where $a_c = 2\langle a \rangle$ and $\phi_{\pm} = \phi_{vac} \pm \sqrt{3\langle(\phi - \phi_{vac})^2\rangle}$ are the classical turning points for the potentials a^2 and (10) respectively. The average values $\langle a \rangle$ and $\langle(\phi - \phi_{vac})^2\rangle$ in the state ψ_E (11) are equal to

$$\langle a \rangle = \frac{\sqrt{2N+1}}{2}, \quad \langle(\phi - \phi_{vac})^2\rangle = \frac{M}{6m^2\langle a \rangle^3}. \quad (15)$$

One can see that the average value $\langle a \rangle \gg 1$ corresponds to the state with $n \gg 1$. Using the wavefunction (11) for the energy density with the potential (10) we obtain

$$\bar{\rho} = \gamma \frac{M}{\langle a \rangle^3} + \frac{E}{\langle a \rangle^4}, \quad (16)$$

where the coefficient $\gamma = 193/12$ arises in calculation of expectation value for the operator of energy density of scalar field and takes into account its kinetic and potential terms. In matter dominated universe $M \gg E/(4\langle a \rangle)$ and from Eqs. (11) and (16) it follows that the quantity of matter/energy M and the mean energy density $\bar{\rho}$ in the universe taken as a whole (i.e. in quantum states which describe only homogenized properties of the universe) satisfy the relations

$$M = \langle a \rangle, \quad \bar{\rho} = \frac{\gamma}{\langle a \rangle^2} \quad (17)$$

which agree with the relations (1) and (2). Substitution of Eq. (17) into (7) leads to the density parameter $\Omega = \bar{\rho}/H^2$, where H^2 coincides with the critical energy density in dimensionless units being used, equal to $\Omega = 1.066$. It means that the universe in highly excited states is spatially flat (to within about 7%). This value of Ω agrees with existing astrophysical data for the present-day universe: $\Omega = 1 \pm 0.12$ [25], $\Omega = 1.02 \pm 0.06$ [26], $\Omega = 1.04 \pm 0.06$ [27], $\Omega = 0.99 \pm 0.12$ [28]. For other values of Ω see e.g. Ref. [29].

4 Coordinate distance to source

From Eqs. (7) and (17) we find the Hubble constant as a function of cosmological redshift $z = a_0/\langle a \rangle - 1$

$$H(z) = H_0(1+z). \quad (18)$$

Let us find the dimensionless coordinate distance $H_0 r(z)$ to source at redshift z , where $r(z)$ is given by

$$\begin{aligned} r(z) &= a_0 \sin \left(\frac{1}{a_0} \int_0^z \frac{dz}{H(z)} \right) \quad \text{for } \Omega > 1, \\ r(z) &= \int_0^z \frac{dz}{H(z)} \quad \text{for } \Omega = 1, \\ r(z) &= a_0 \sinh \left(\frac{1}{a_0} \int_0^z \frac{dz}{H(z)} \right) \quad \text{for } \Omega < 1. \end{aligned} \quad (19)$$

It is connected with a luminosity distance d_L by a simple relation $r(z) = (1+z)^{-1} d_L$ (see Refs. [3, 7, 30, 31]). For a flat universe with the Hubble constant (18) the dimensionless coordinate distance obeys the logarithmic law

$$H_0 r(z) = \ln(1+z). \quad (20)$$

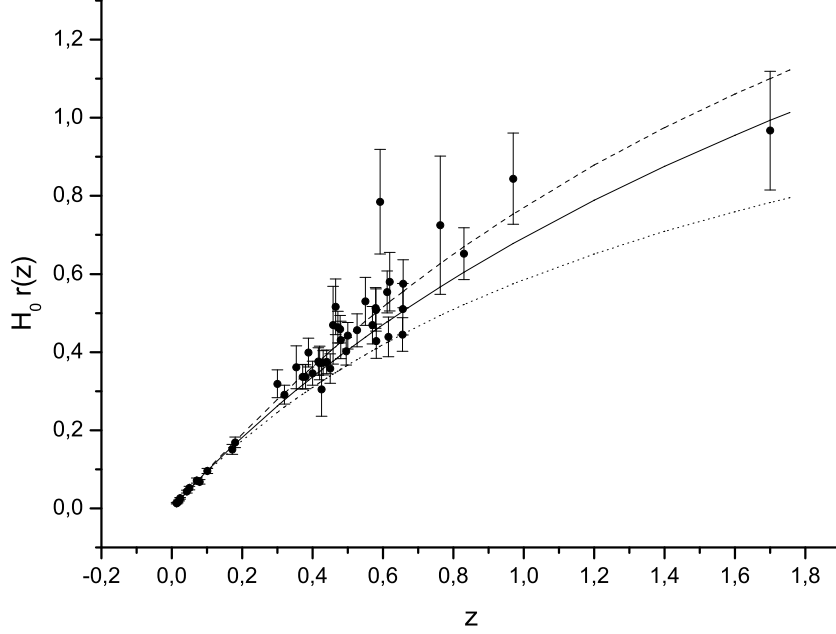


Figure 1: Dimensionless coordinate distances $H_0 r(z)$ to supernovae at redshift z . The observed SNe Ia are shown as solid circles. The model (20) is drawn as a solid line. The Λ CDM-model with $\Omega_M = 0.3$ and $\Omega_X = 0.7$ is represented as a dashed line. The model with $\Omega_M = 1$ is shown as a dotted line.

The dimensionless coordinate distances to the SNe Ia and RGs obtained in Ref. [30] from the observational data (solid circles and boxes) and our result (20) (solid line) are shown in Figs. 1 and 2. The Λ CDM-model with $\Omega_M = 0.3$ (matter component) and $\Omega_X = 0.7$ (dark energy in the form of cosmological constant) and the model without dark energy ($\Omega_M = 1$) are drawn for comparison. Among the supernovae shown in Figs. 1 there are the objects with central values of coordinate distances which are better described by the Λ CDM-model (e.g. 1994am at $z = 0.372$; 1997am at $z = 0.416$; 1995ay at $z = 0.480$; 1997cj at $z = 0.500$; 1997H at $z = 0.526$; 1997F at $z = 0.580$), the law (20) (e.g. 1995aw at $z = 0.400$; 1997ce at $z = 0.440$;

1995az at $z = 0.450$; 1996ci at $z = 0.495$; 1996cf at $z = 0.570$; 1996ck at $z = 0.656$) and the model with $\Omega_M = 1$ (1994G at $z = 0.425$; 1997aj at $z = 0.581$; 1995ax at $z = 0.615$; 1995at at $z = 0.655$). The RG data [30] demonstrate the efficiency of the model (17) as well (Fig. 2). The quantum model predicts the coordinate distance to SN 1997ff at $z \sim 1.7$ which is very close to the observed value (see Fig. 1). In the range $z \leq 0.2$ three above mentioned models give in fact the same result.

The density $\bar{\rho}$ (16) contains all possible matter/energy components in the universe. Let us separate in (16) the baryon matter density equal to $\Omega_B \approx 0.04$ [14, 29] which makes a small contribution to the matter density $\Omega_M \approx 0.3$ [6]. If we assume that the baryon density varies as $\rho_B \sim a^{-3}$, while the remaining constituents of density effectively decrease as a^{-2} , then the value $H_0 r(z)$ calculated in such a model will practically coincide with the coordinate distance shown in Fig. 1 as a solid line.

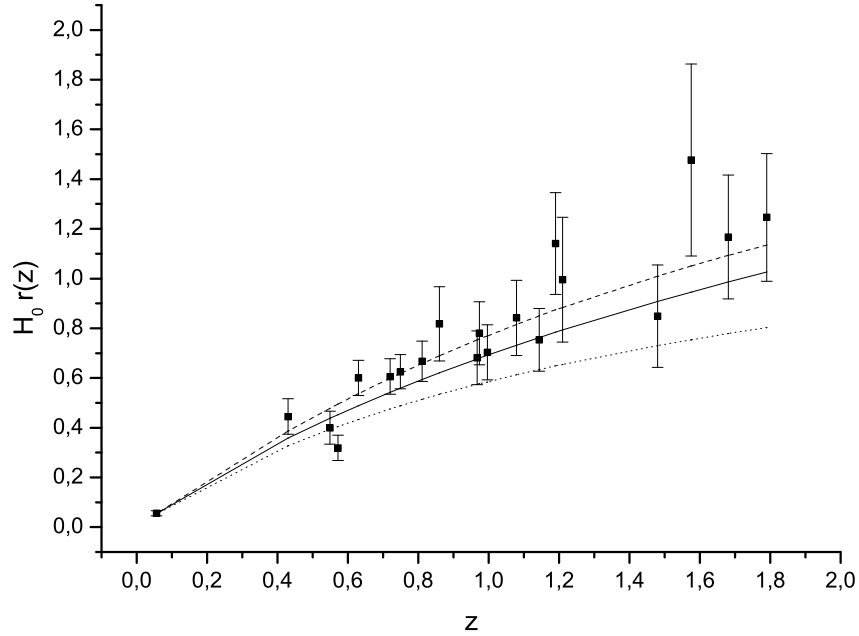


Figure 2: Dimensionless coordinate distances $H_0 r(z)$ to radio galaxies at redshift z . Radio galaxies are shown as solid boxes. The rest as in Fig. 1.

In Ref. [7] a conclusion is drawn that the model of dark energy with $w_X = -\frac{1}{3}$ implying $\rho_X \sim a^{-2}$ agrees with the recent CMB observations made by WMAP as well as with the high redshift supernovae Ia data. Such a universe is decelerating. In our quantum model the total dark matter/energy in the states which describe only

homogenized properties of the universe varies effectively as a^{-2} . In terms of general relativity it means that its negative pressure compensates for action of gravitational attraction and the universe as a whole expands at a steady speed.

5 Quantum fluctuations of scale factor

Deviations of $H_0 r(z)$ from the law (20) towards both larger and smaller distances for some supernovae can be explained by the local manifestations of quantum fluctuations of scale factor about the average value $\langle a \rangle$ which arose in the Planck epoch ($t \sim 1$) due to finite widths of quasistationary states. As it is shown in Refs. [15, 16] such fluctuations can cause the formation of nonhomogeneities of matter density which have grown with time into the observed large-scale structures in the form superclusters and clusters of galaxies, galaxies themselves etc. Let us consider the influence of mentioned fluctuations on visible positions of supernovae.

The position of quasistationary state E_n can be determined only approximately, $E_n \rightarrow E_n + \delta E_n$, where $|\delta E_n| \sim \Gamma_n$, Γ_n is the width of the state. The scale factor of the universe in the n -th state can be found only with uncertainty,

$$\langle a \rangle \rightarrow \langle a \rangle + \delta a, \quad (21)$$

where the deviation $\delta a \geq 0$ is determined by both the value δE_n and the time of its formation [15, 16]. Since Γ_n is exponentially small for the states $n \gg 1$, the fluctuations δE_n in the early universe are the main source for δa . The calculations demonstrate that the lowest quasistationary state has the parameters $E_{n=0} = 2.62$ and $\Gamma_{n=0} = 0.31$ (in dimensionless units). The radius of curvature is $\langle a \rangle_{n=0} \sim 1$, while the lifetime of such a universe is $\tau \sim \Gamma_{n=0}^{-1} \sim 3$. Within the time interval $\Delta t \leq 3$ the nonzero fluctuations of scale factor with relative deviation equal e.g. to

$$\begin{aligned} \left| \frac{\delta a}{\langle a \rangle} \right| &\lesssim 0.022 \quad \text{at } \Delta t = 1, \\ \left| \frac{\delta a}{\langle a \rangle} \right| &\lesssim 0.040 \quad \text{at } \Delta t = 2, \\ \left| \frac{\delta a}{\langle a \rangle} \right| &\lesssim 0.077 \quad \text{at } \Delta t = 3 \end{aligned} \quad (22)$$

can be formed in the universe (see Appendix B). Such fluctuations of the scale factor cause in turn the fluctuations of energy density which can result in formation of structures with corresponding linear dimensions under the action of gravitational attraction. For example, for the current value $\langle a \rangle \sim 10^{28}$ cm the dimensions of large-scale fluctuations $\delta a \lesssim 70$ Mpc, $\delta a \lesssim 120$ Mpc, and $\delta a \lesssim 200$ Mpc correspond to relative deviations (22). On the order of magnitude these values agree with the scale of superclusters of galaxies.

If one assumes that just the fluctuations δa cause deviations of positions of sources at high redshift from the law (20), then it is possible to estimate the values of relative deviations $\delta a/\langle a \rangle$ from the observed values $H_0 r(z)$. The fluctuations of scale factor (21) generate the changes of coordinate distances,

$$H_0 r(z) = \ln \left[\left(1 + \frac{\delta a}{\langle a \rangle} \right)^{-1} (1 + z) \right]. \quad (23)$$

The possible values of coordinate distances obtained from Eq. (23) for the relative deviations (22) are shown as a shaded area in Fig. 3. Practically all supernovae in this redshift interval fall within the limits of (22). The only exception is SN 1997K at $z = 0.592$ which should be characterized by too sharp negative relative deviation $\delta a/\langle a \rangle = -0.274$ (for central value) even in comparison with the largest possible fluctuations of the scale factor.

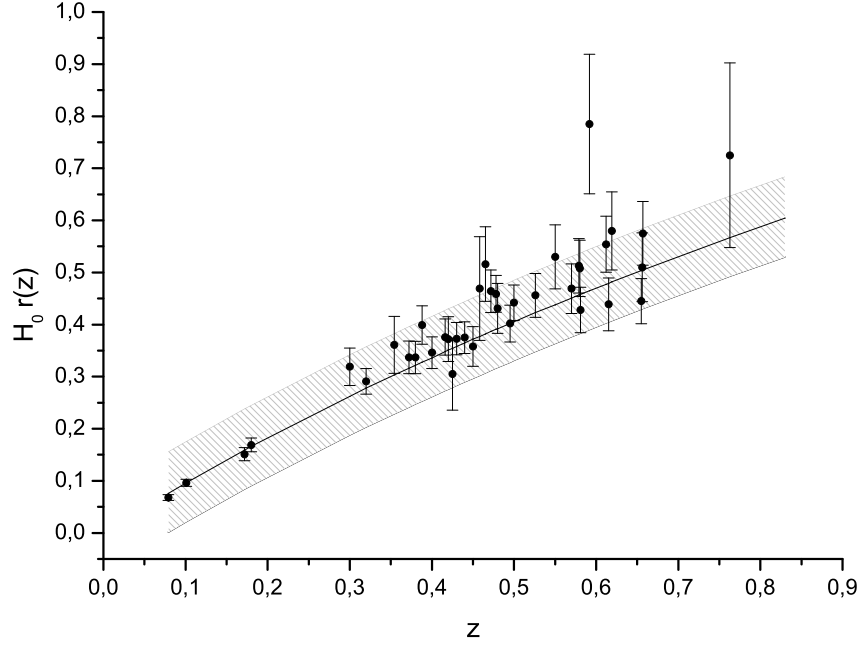


Figure 3: Dimensionless coordinate distances $H_0 r(z)$ to supernovae in the interval $z = 0.1 - 0.8$. A shaded area corresponds to possible values of coordinate distances in the model (23) for the relative deviations (22). The rest as in Fig. 1.

The same analysis one can make for RGs as well.

Thus the observed faintness of the SNe Ia can in principle be explained by the logarithmic-law dependence of coordinate distance on redshift in generalized form (23) which takes into account the fluctuations of scale factor about its average value. These fluctuations can arise in the early universe and grow with time into observed deviations of the coordinate distances of separate SNe Ia at the high redshift. They produce accelerating or decelerating expansions of space subdomains containing such sources whereas the universe as a whole expands at a steady rate.

6 Some cosmological consequences

In matter dominated universe from Eqs. (7) and (17) it follows that

$$\langle a \rangle \sim t, \quad Ht = 1 \quad (24)$$

for any value of z . The first relation agrees with the astrophysical data (1) for the present-day universe. The dimensionless age parameter is known in the range $0.72 \lesssim H_0 t_0 \lesssim 1.17$ with the central value $H_0 t_0 \approx 0.89$ [4]. The other values $H_0 t_0 = 0.96 \pm 0.04$ [6] and $H_0 t_0 \approx 0.93$ [14] agree with the theoretical prediction (24) as well.

In radiation dominated universe from Eq. (7) at fixed E it follows the “standard” expression for the scale factor (see e.g. Ref. [31])

$$\langle a \rangle = \left(2\sqrt{Et} \right)^{1/2}, \quad (25)$$

where the time t is counted from the singular state with zero value of the scale factor. The solution (25) is the special case of the solution given in the Appendix B. In general relativity and in quantum theory the evolution of the universe is described differently. In general relativity a scale factor is a function of time, e.g. as in (25). The quantum model is based on the time-independent equation (3). And the evolution is described as successive transitions of the universe from one state (e.g. with number n) to another (n'). From the first equation (15) it follows that such transitions will manifest themselves in the form of expansion ($n' > n$) or contraction ($n' < n$) of the universe. Direct calculations [15, 16] and physical arguments [23, 24] show that the $n \rightarrow n' = n + 1$ transitions are the most probable. As a result the universe can get into the state with $\langle a \rangle \sim \sqrt{n} \gg 1$ in a finite time.

In the quantum model every state of the universe is characterized by its own eigenvalue E . In the epoch, when the contribution from matter/energy in the form of elementary quantum excitations of the vibrations of the field ϕ into the mean total energy density can be neglected (in comparison with the contribution from the relativistic matter), $M \ll E/(4\langle a \rangle)$, the universe as a whole, according to (11), is characterized by the parameter $E \sim \langle a \rangle^2$ and the energy density $\bar{\rho} \approx E/\langle a \rangle^4$ will effectively decrease as $\langle a \rangle^{-2}$. Then the solution of Eq. (7) will have the same form (24) as in subsequent matter dominated universe. The same result can be

formally obtained directly from Eq. (25) as well if one assumes that in accordance with quantum mechanical treatment it describes the state in which the eigenvalue $E \sim \langle a \rangle^2$ corresponds to given $\langle a \rangle$. Since in quantum description after transition from radiation dominated to matter dominated universe the effective density $\bar{\rho}$ still decreases as $\langle a \rangle^{-2}$, then the law of expansion also does not change. The linear dependence of $\langle a \rangle$ on t as in Eq. (24), if it is realized in the universe during long enough period of time (possibly with the exception of short epoch in history of the early universe when $\langle a \rangle \sim 1$), allows to solve the old problems of standard cosmology (e.g. the problems of flatness, horizon, and age [12, 13]) without appeal to the hypothesis about de-Sitter (exponential) stage of expansion of the early universe [13, 22].

In addition to the prediction about the steady-speed expansion of the universe as a whole (at the same time the accelerating or decelerating motions of its sub-domains remain possible on a cosmological scale as it is shown in Sec. 5 of this paper) the quantum model allows an increase of quantity of matter/energy in matter dominated universe according to (11). If the mass m of elementary quantum excitations of the vibrations of the field ϕ remains unchanged during the expansion of the universe, then the increase of M can occur due to increase in number s of these excitations. But the increase in s does not mean that a quantity of observed matter in some chosen volume of the universe increases. According to the model proposed in Refs. [23, 24] the observed “real” matter (both luminous and dark) is created as a result of the decay of elementary quantum excitations of the vibrations of the field ϕ (under the action of gravitational forces) into baryons, leptons and dark matter. The undecayed part of them forms what can be called a dark energy. Such a decay scheme leads to realistic estimates of the percentage of baryons, dark matter and dark energy in the universe with $\langle a \rangle \gg 1$ and $M \gg 1$. Despite the fact that the quantity of matter/energy can increase, the mean total energy density decreases and during the expansion of the universe mainly the number of elementary quantum excitations of the vibrations of the field ϕ increases. Their decay probability is very small, so that basically only the dark energy is created. These circumstances can explain the absence of observed events of creation of a new baryonic matter on a cosmologically significant scale.

The proposed approach to the explanation of observed dimming of some SNe Ia may provoke objections in connection with the problem of large-scale structure formation in the universe, since the energy density $\bar{\rho}$ in the form (17) cannot ensure an existence of a growing mode of the density contrast $\delta\bar{\rho}/\bar{\rho}$ (see e.g. Refs. [29, 31, 32]). As we have already mentioned above in Sec. 3 of this paper the density $\bar{\rho}$ (17) describes only homogenized properties of the universe as a whole. It cannot be used in calculations of fluctuations of energy density about the mean value $\bar{\rho}$. Under the study of large-scale structure formation one should proceed from the more general expression for the energy density (16). Defining concretely the contents of matter/energy M , as for instance in the model of creation of matter mentioned

above, one can make calculations of density contrast as a function of redshift. The problem of large-scale structure formation is one of the main problems of cosmology (see e.g. Refs. [29, 33]). It goes beyond the tasks of this paper and requires a special investigation. The ways of its solution in the quantum model are roughly outlined in Ref. [16].

A Appendix: Derivation of Eq. (7)

The time-dependent equation which describes the quantum model of the homogeneous, isotropic and spatially closed universe has a form [15, 16]

$$i \partial_T \Psi = \hat{\mathcal{H}} \Psi, \quad (\text{A1})$$

where

$$\hat{\mathcal{H}} = \frac{1}{2} \left(\partial_a^2 - \frac{2}{a^2} \partial_\phi^2 - a^2 + a^4 V(\phi) \right) \quad (\text{A2})$$

is a Hamiltonian-like operator. The wavefunction Ψ depends on a scale factor a , scalar field ϕ , and time coordinate T . In derivation of Eq. (A1) the time T is introduced as an additional (embedding) variable which describes a motion of a source in a form of relativistic matter of an arbitrary nature. It is related to the synchronous proper time t by the differential equation: $dt = a dT$ [15]. Eq. (A1) allows a particular solution with separable variables

$$\Psi = e^{\frac{i}{2}ET} \psi_E, \quad (\text{A3})$$

where the function ψ_E satisfies the time-independent equation (3). The general solution of Eq. (A1) has a form of the superposition of the states (A3) with some weighting function which characterizes the distribution in E of the states at the instant $T = 0$ [16].

Using Eq. (A1) and taking into account that the Hamiltonian (A2) is Hermitean we obtain the equation which determines a change in time T of the average value of the physical quantity A

$$\frac{d}{dT} \langle \hat{A} \rangle = \frac{1}{i} \langle [\hat{A}, \hat{\mathcal{H}}] \rangle + \langle \partial_T \hat{A} \rangle, \quad (\text{A4})$$

where the operator \hat{A} corresponds to A , $[\hat{A}, \hat{\mathcal{H}}] = \hat{A}\hat{\mathcal{H}} - \hat{\mathcal{H}}\hat{A}$, and angle brackets denote the average value in the state Ψ . Introducing the operator $d\hat{A}/dT$ by the relation

$$\left\langle \frac{d\hat{A}}{dT} \right\rangle = \frac{d}{dT} \langle \hat{A} \rangle, \quad (\text{A5})$$

Eq. (A4) can be rewritten in the operator form

$$\frac{d\hat{A}}{dT} = \frac{1}{i} [\hat{A}, \hat{\mathcal{H}}] + \partial_T \hat{A}. \quad (\text{A6})$$

Setting $\hat{A} = \hat{a}$, and using the explicit form of the Hamiltonian (A2) we find

$$a \frac{d\hat{a}}{dt} = -\hat{\pi}_a, \quad (\text{A7})$$

where $\hat{\pi}_a = -i \partial_a$ is the momentum operator canonically conjugate with a , and the operator $\hat{a} = a$. The operator equation (A7) is equivalent to the definition of the momentum $\pi_a = -a da/dt$ canonically conjugate with the variable a in classical cosmology [13, 16]. Using (A7) we obtain

$$\left\langle -\frac{1}{a^4} \partial_a^2 \right\rangle = \left\langle \left(\frac{1}{a} \frac{d\hat{a}}{dt} \right)^2 \right\rangle. \quad (\text{A8})$$

Let us represent the operators \hat{a} and $d\hat{a}/dt$ as follows:

$$\hat{a} = \langle a \rangle + \hat{\xi}, \quad \frac{d\hat{a}}{dt} = \frac{d\langle a \rangle}{dt} + \frac{d\hat{\xi}}{dt}, \quad (\text{A9})$$

where generally speaking the time derivative of the operator $\hat{\xi}$ is not equal to $d\hat{\xi}/dt$. Then

$$\left\langle \left(\frac{1}{a} \frac{d\hat{a}}{dt} \right)^2 \right\rangle = \left\langle \left(1 + \frac{\hat{\xi}}{\langle a \rangle} \right)^{-2} \left(1 + \frac{d\hat{\xi}}{d\langle a \rangle} \right)^2 \left(\frac{1}{\langle a \rangle} \frac{d\langle a \rangle}{dt} \right)^2 \right\rangle. \quad (\text{A10})$$

In a first approximation one can neglect the deviation of a from its average value $\langle a \rangle$ and set $a = \langle a \rangle$. Then using Eqs. (A8) and (A10) and taking into account that a and ϕ are independent variables, Eq. (6) can be reduced to the form (7).

Let us note that setting $\hat{A} = \hat{\pi}_a$ from Eq. (A6) we obtain

$$a \frac{d\hat{\pi}_a}{dt} = \frac{2}{a^3} \hat{\pi}_\phi^2 + a - 2a^3 V(\phi), \quad (\text{A11})$$

where $\hat{\pi}_\phi = -i \partial_\phi$ is the momentum operator canonically conjugate with ϕ . From (A11) it follows the second Einstein-Friedman equation for average values. Similarly setting $\hat{A} = \phi$ and $\hat{A} = \hat{\pi}_\phi$ one can obtain equations which describe an evolution in time of the field ϕ .

B Appendix: Fluctuations of scale factor

In order to estimate an amplitude of fluctuations of scale factor (relative deviation $\delta a / \langle a \rangle$) we shall consider the solution of Eq. (7) for average values in the epoch when the matter is represented by a slow-roll scalar field and the kinetic term in density (8) can be neglected. In this case from Eq. (7) we obtain

$$\begin{aligned} \langle a \rangle &= \left\{ \frac{1}{2V_n} \left[1 + (2V_n \alpha^2 - 1) \cosh \left(2\sqrt{V_n} \Delta t \right) \right] \right. \\ &\quad \left. + \sqrt{\frac{E'}{V_n}} \sinh \left(2\sqrt{V_n} \Delta t \right) \right\}^{1/2}, \end{aligned} \quad (\text{B1})$$

where $V_n = \langle V \rangle$ depends on a state of field ϕ , $E' = E_n - \alpha^2 + \alpha^4 V_n$, and $\Delta t = t - t_{\text{initial}}$. The solution (B1) corresponds to the boundary condition $\langle a \rangle = \alpha$ at $t = t_{\text{initial}}$. For $2\sqrt{V_n}\Delta t \ll 1$ we have

$$\langle a \rangle = \left[\alpha^2 + 2\sqrt{E'}\Delta t \right]^{1/2}. \quad (\text{B2})$$

Keeping main terms only from (B1) we find the following expression for the amplitude of fluctuations

$$\frac{\delta a}{\langle a \rangle} = \frac{\frac{1}{4} \frac{\delta E'}{E'}}{1 + \frac{1}{2} \sqrt{\frac{V_n}{E'}} \left[\left(\alpha^2 - \frac{1}{V_n} \right) \tanh(\sqrt{V_n}\Delta t) + \alpha^2 \coth(\sqrt{V_n}\Delta t) \right]}. \quad (\text{B3})$$

For the lowest state with the parameters $E_{n=0} = 2.62$, $\delta E' \approx \Gamma_{n=0}/2 = 0.16$, $V_{n=0} = 0.08$, and $\alpha \approx 1$ [16] from (B3) we find the values of relative deviations (22).

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